Geometry of Power Flow in Smart Transmission Grid: Reliability, Congestion and Self-Heals

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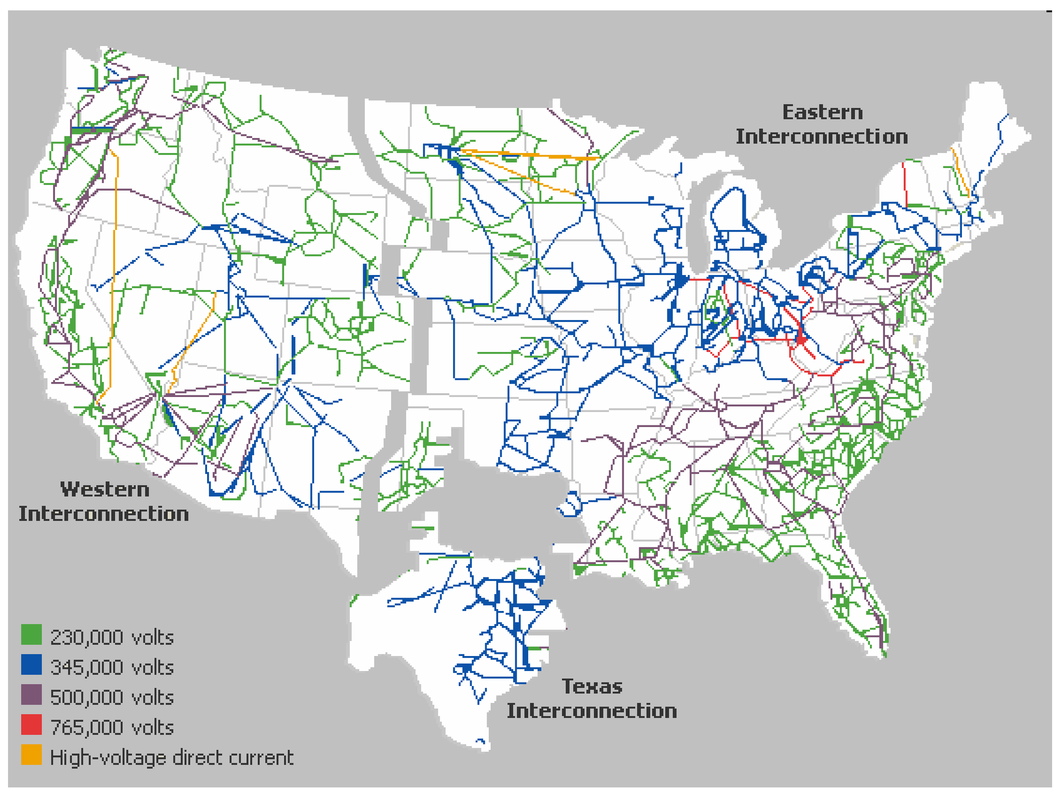
Smart Transmission **is an essential, or maybe the most essential, pillar for the** ***Smart Grid***



Evolution to more reliable, safe and secure grid

Wide use of stochastic renewable energy resources

Bulk power transmission over long distances

Distribution within congested areas and mega cities

Future smart bulk transmission system must be resilient to larger power fluctuations, more distributed malfunctions, and likely attacks

Increasing competition in energy trades

Reverse power flows

Increasing grid size and complexity

To analyze this large complex network we need to use all The tools at our disposal.

Time and again, we have been pushed beyond the limit of the existing techniques and have had to create new and better tools.

Contribution: Inspired from Riemannian geometry, we investigate the reliability of power grid from the topology of its hidden metric space.

Key claim: Extreme load at specific parts of a large power grid can occur as a consequence of the local negative curvature in its hidden metric space.

Intellectual merits:
1) We draw a new course in the topological study of power grid.
2) We extend the Riemannian geometry metaphor to the power grid.
3) We address a unifying approach to deal with power and data networks.
4) We provide an analytical measure for the criticality of lines and stations, applicable to reliability assessment and flow control.

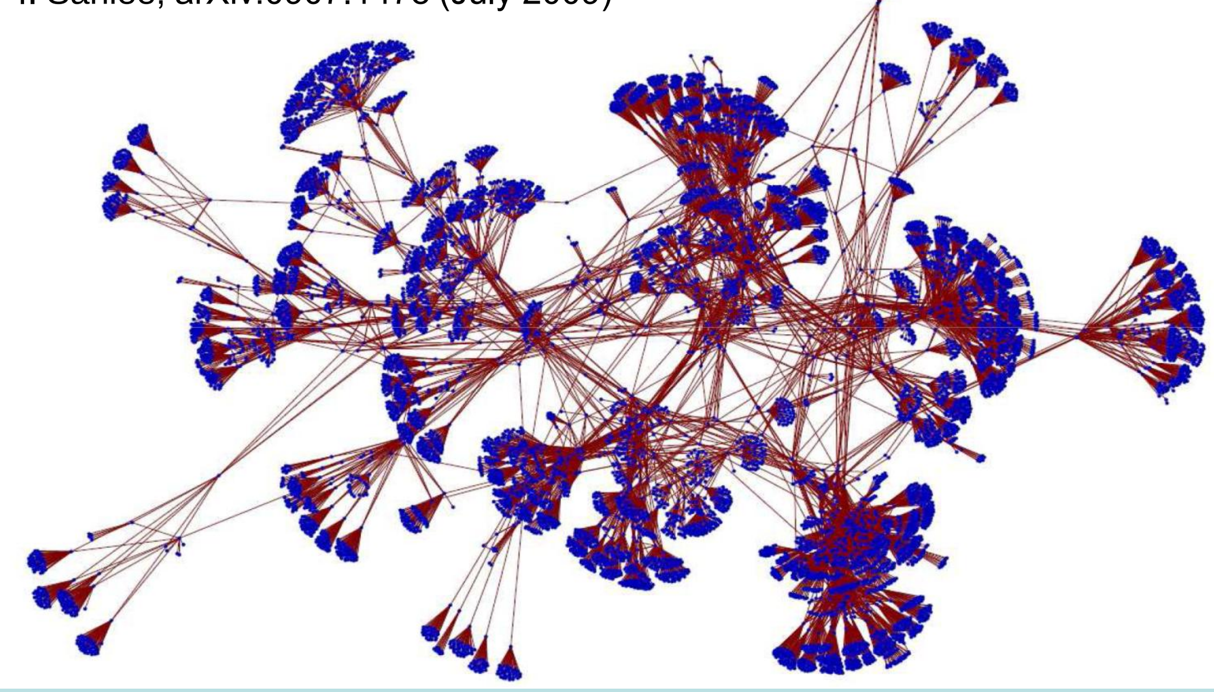
Large Scale Networks

- Hard to visualize due to scale
- Hard to realize the essential characteristics for overall functionality, reliability and security
- Is there a way to summarize the critical network information?

Under some conditions, the network can be approximated by a Riemannian manifold. Then a promising direction is to look at two main fundamental features of this manifold:

Dimension & Curvature

Rocketfuel database AT&T with 10152 nodes and 28638 links, as described in "The Large Scale Curvature of Networks, by O. Narayan, I. Saniee, arXiv:0907.1478 (July 2009)"

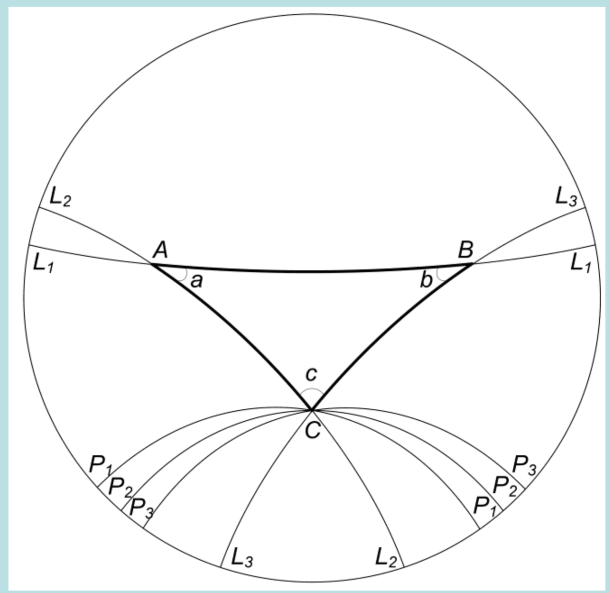


Local Curvature vs. Curvature in the Large

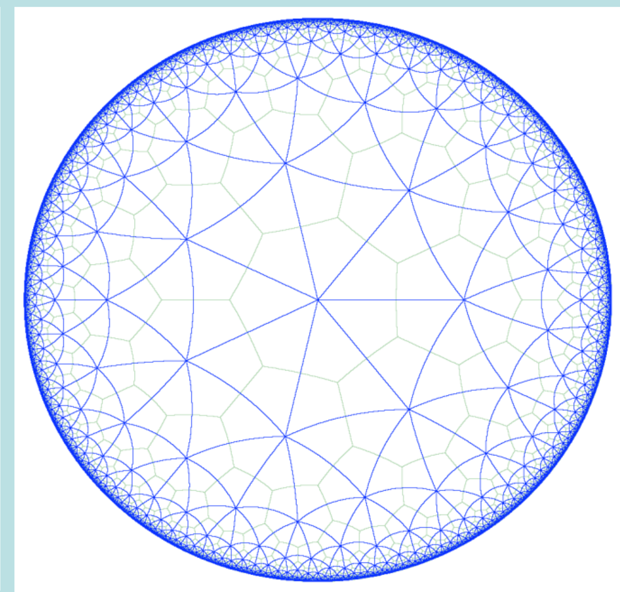
Gromov Thin Triangle Condition allows to determine whether a graph is negatively curved in the very large scale.

For a triangle drawn on a negatively curved surface, the sum of the angles is less than π, giving it a thin appearance.

Scaled Gromov Property provides us with an extension to some medium scale, and by the same token to the concept of nonnegatively curved graphs.



Hyperbolic lines intersect to form triangle ABC. The sum of its angles $a+b+c<\pi$. There are infinitely many lines parallel to line L1 and go through a point C that does not belong to L1.



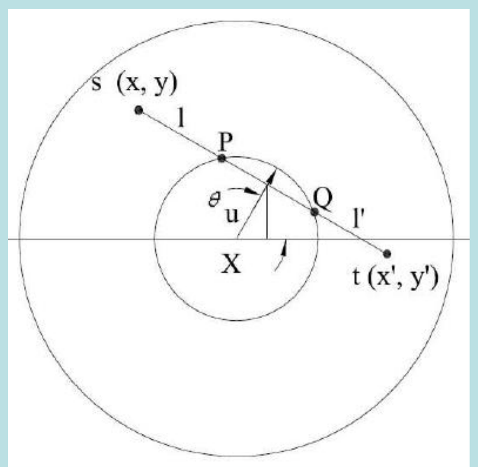
Tessellation of the hyperbolic plane. All triangles and heptagons are of the same hyperbolic size but the size of their Euclidean representations exponentially decreases.

Moment of Inertia

The moment of inertia of a weighted graph with respect to a node x is defined by $\phi(x)=(1/\lambda)\sum_{x_i} d^{\alpha}(x,x_i)$, for some constants $\alpha>1$ and $\lambda>0$.

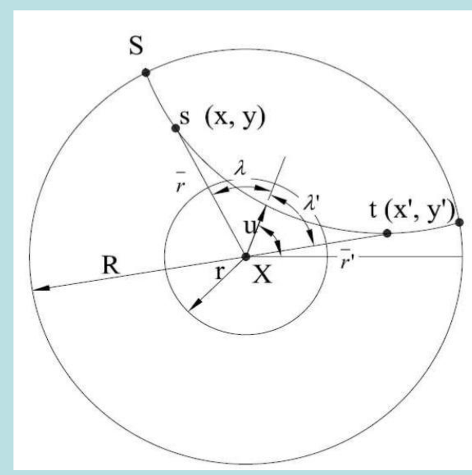
Negatively Curved Network

Consider a large, but finite, negatively curved graph, subject to uniformly distributed demand for commodities. Then, there are some specific nodes with very high traffic, which are the ones of least moment of inertia. If the graph is non-negatively curved, then both the traffic and inertia are more evenly distributed than in the case of a negatively curved graph. Furthermore, if the graph is positively curved with enough symmetry, both the traffic and inertia are uniformly distributed.



Traffic in Euclidean disk

The source $s(x,y)$ sends commodities to the destination $t(x,y)$ along a straight line that enters the small ball X along PQ. The traffic load is the average length of PQ. The optimal paths are uniformly distributed



Traffic in the Poincare disk

The source to target geodesic [$s(x,y)$, $t(x',y')$] arches towards centre so that its average length in the small ball $X=Br(0)$ is considerably larger than in the Euclidean case. The optimal paths are maximally distributed at the center.

Challenges in Modeling Power with Traffic

- Traffic needs to be expressed by a simple variable as the rate of commodities passing through a node or a link. However, electrical power requires two variables to be identified, a generalized coordinate (charge) and a generalized force (voltage).
- A commodity like a message has a specific header and is transferred from source to destination through an optimal path. However, electrical power flows along all transmission lines and stations from generating source to consuming loads.

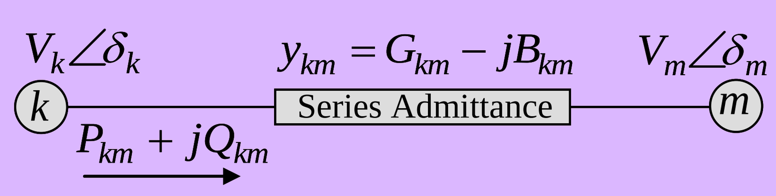
Resistive Graph vs. Transmission Grid

Definition: The virtual resistive grid associated with a specific steady-state power flow condition is defined as a resistive graph *isomorphic* to the power grid, in which the resistance of each link is equal to $R_{km}=1/B_{km}\bar{V}_k\bar{V}_m\cos\bar{\theta}_{km}$.

- All variables of the system, when evaluated in a specific steady-state mode, are assumed constant in short-term analysis, even though their values can change in medium- and long-term operation.
- The root of fluctuations in the system is the variation of power supply/demand, happening in buses. Then, deviation of voltage magnitude and phase angle will be the consequence of those variations. Accordingly, the virtual resistive grid will only have current sources in its nodes as metaphors for the fluctuation of bus complex powers, where the stationary value of each current source is zero, corresponding to no power fluctuation in the bus supply/demand.

Theorem: Consider the virtual resistive grid associated with a specific steady-state mode. Let a set of complex current sources ψ_k injecting into the nodes, resulting in a set of node complex voltages U_k and a set of link complex currents J_k. Then, the fluctuation of line complex power and bus complex voltage in the power grid satisfy $\tilde{P}_{km}+j\tilde{Q}_{km}=J_{km}$ and $\tilde{\theta}_k+j\tilde{V}_k/\bar{V}_k=U_k$, iff the fluctuation of net complex power injection into the buses satisfy $\tilde{S}_k=\psi_k$.

Power Flow Equations

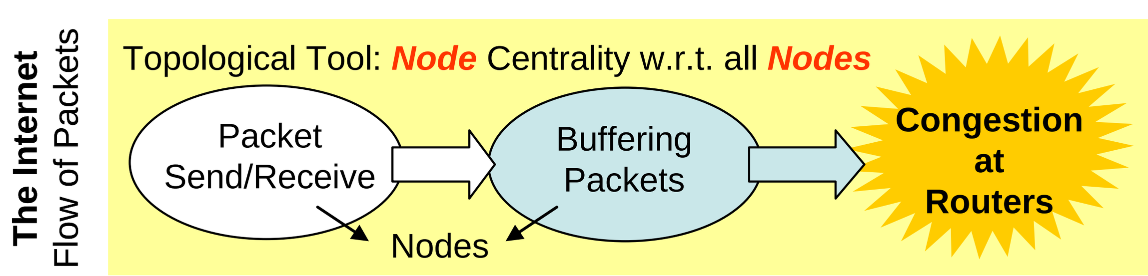


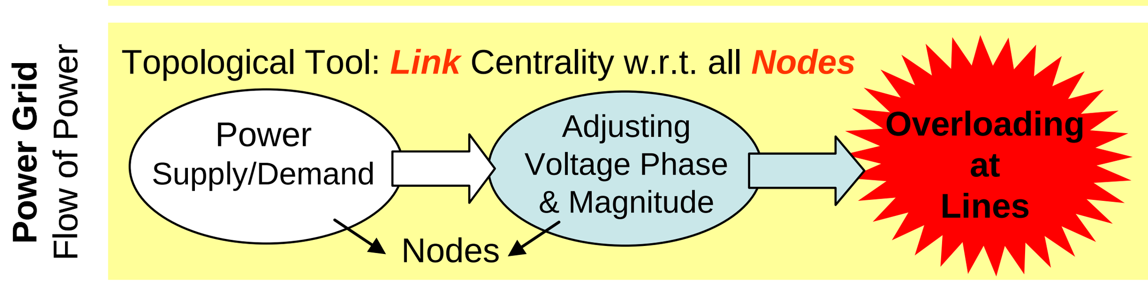
$$\begin{cases} P_{km}=G_{km}V_k^2-G_{km}V_kV_m\cos(\theta_k-\theta_m)+B_{km}V_kV_m\sin(\theta_k-\theta_m) \\ Q_{km}=B_{km}V_k^2-B_{km}V_kV_m\cos(\theta_k-\theta_m)-G_{km}V_kV_m\sin(\theta_k-\theta_m) \\ S_k=\sum_m P_{km}+j\sum_m Q_{km}=E_kI_k^* \end{cases}$$

$$\begin{cases} \tilde{P}_{km}\cong\left(B_{km}\bar{V}_k\bar{V}_m\cos\bar{\theta}_{km}\right)\left(\tilde{\theta}_k-\tilde{\theta}_m\right) \\ \tilde{Q}_{km}\cong\left(B_{km}\bar{V}_k\bar{V}_m\cos\bar{\theta}_{km}\right)\left(\tilde{V}_k/\bar{V}_k-\tilde{V}_m/\bar{V}_m\right) \\ \tilde{S}_{km}\cong\tilde{P}_{km}+j\tilde{Q}_{km} \end{cases}$$

Contrast between Packet Congestion and Power Overloading

- In a data network, limitation is on routers with packet drops, whereas in power grid limitation is on lines with overload trips.
- In a data network both send/receive and congestion occur in nodes, whereas in power grid supply/demand occurs in buses (nodes) but overloading happens in lines (links).





Weighted Electrical Centrality (Inverse Inertia)

Assume a current source I_l is injected into node x_l in the virtual resistive grid. Link x_kx_m receives a current i_{km}, and $0\le|i_{km}/I_l|\le 1$ represents a measure of electrical closeness between line x_kx_m and bus x_l. Weighted electrical centrality for a link x_kx_m is defined by the sum of the weighted closeness between this link and all nodes in the virtual resistive grid, i.e.,

$$C_{km}(x_kx_m)=\frac{1}{(N-1)\cdot R_{km}}\sum_{x_i}\left(\left(\mathcal{L}_0^{-1}\right)_{ki}-\left(\mathcal{L}_0^{-1}\right)_{mi}\right)\cdot|S_i|\cdot\rho_i$$

where $|S_i|$ is the bus net apparent power, $0\le\rho_i\le 1$ is the bus operation risk factor, and N is the number of nodes in the grid. Then, the normalized moment of inertia of the power grid with respect to a line is defined as

$$\phi_{km}(x_kx_m)=\left(1-\frac{C_{km}(x_kx_m)}{\max_{x_ix_j} C_{ij}(x_ix_j)}\right)^{\alpha}$$

for a constant $\alpha>1$.

Hyperbolic Resistive Grid

Theorem: If the resistive graph is Gromov hyperbolic, then for any pair of nodes x_k, x_l, we have

$$R_{eff}(x_k,x_l)=O\left(R(x_k,x_l)\right)$$

Negatively Curved Power Grid

Corollary1: Consider the virtual resistive grid associated with a specific steady-state mode of the power grid with a set of bus operation risk factors. If this resistive grid is Gromov hyperbolic, then under uniform distribution of power fluctuations in supply/demand, the lines with least moments of inertia experience most fluctuations in their transmission load.

Reliable Transmission

Corrollary2: Consider the negatively curved power grid in Corollary 1. Then for a reliable power transmission under uniform distribution of power fluctuations in supply/demand, highest free capacity must be allocated to the lines with lowest moments of inertia.

Hierarchical Reliability Analysis

Long-term Operation
- Generation Resource Planning
- Transmission Planning

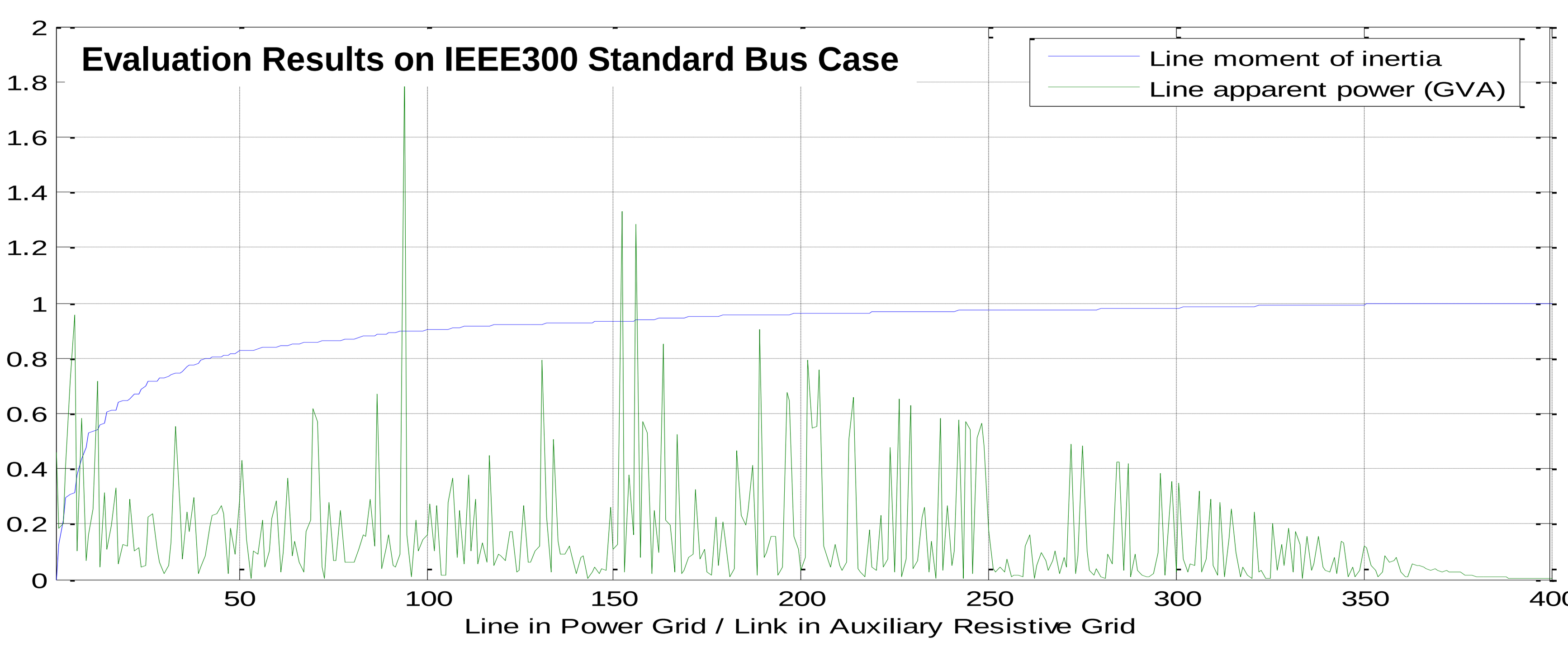
Mid-term Operation
- Maintenance Scheduling
- Fuel Allocation
- Emission Allowance
- Optimal Operation Cost

Short-term Operation
- Reliable Power Generation
- Topology-constrained Optimal Power Flow

Real-time Reliability Assessment
- SCADA / System Monitoring
- Contingency Analysis

Remark:

If a line is in high utilization, the red flag is already raised, even in a traditional dispatch. However, the claim here is that for a line with respect to which the power grid has low moment of inertia, the red flag must be raised in quite a lower utilization compared to that in a traditional dispatch. Such a line, even with lower utilization, may be at higher risk of overloading in the presence of disturbance in supply/demand.



(1) The grid is locally negatively curved with respect to some lines and buses. (2) Number 1 line has zero moment of inertia with 458 MVA transmission load. This line is the only one connecting the power grid to its reference bus, namely *swing bus.* (3) Number 0 to 50 lines are in high centrality with respect to the fluctuations of power supply/demand in buses. To have a reliable power grid, these lines must operate quite far away their rated capacities. (4) Number 51 to 180 lines are in medium centrality, where a collection of lines with highest transmitting power occurs.